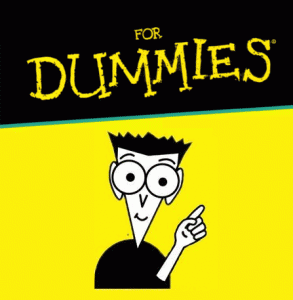


Factoring and Solving Equations in Algebra 2



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Factoring the GCF

Factoring the GCF or greatest common factor out of an expression is similar to doing the reverse of the distributive property.

1)Among the terms of the expression, look for the largest number and/or variable that can be divided among all the terms of the equation.

For example: *12x­6+ 4x4+ 8x8*  
It seems like the coefficients of each term can be divided by *4*.

When the same variables with exponents are divided, the result is the subtraction of the exponents. This means *x*4 is the largest variable that can be factored out.

Together, the greatest common factor is *4x4*.

2) Divide each term of the expression by the GCF.

*4x4* results in *3x2+1+2x4.*

3) Place parentheses around the expression and place the GCF next to the expression

*4x4(3x2+1+2x4)*

Make sure to double check if you have factored completely. If not, simply take out another factor, divide it by all the terms in the parentheses, and multiply what you thought was the GCF by the factor.

Factoring The Difference of Two Perfect Squares

When x-a is multiplied by x + a, the result is x2 -a + a - a2, which simplifies to x2 - a2

Knowing this fact you can take an expression that is the difference of two perfect and squares and find its factors.

1) Example: *16x4- 4x2*

First, make sure the variables and coefficients are perfect squares and the exponents are even and not 0 unless you do not mind having a square root in your answer or fractional exponents.

2) Square root each term.

The square root of *16x4 is 4x2* and the square root of *4x2is 2x.*

3) In parentheses, write the first factor's square root plus the second factor's square root and in another pair of parentheses,

write the first factor's square root minus the second factor's square root.

*(4x2+ 2x)( 4x2- 2x)*

Make sure that when you factor the difference of perfect squares you do not make the careless mistake of factoring the sum of squares.

Factoring Perfect Trinomials

Notice that *(x+y)2 = x2 + 2xy +y2* and *(x-y)2 = x2 - 2xy +y2*.

This means that given a trinomial, if the first and third term are positive, there is a chance that the trinomial is a binomial squared.

If the square root of the first term multiplied by the square root of the third term multiplied by 2 is equivalent to the second term, then the trinomial is definitely the square of a binomial.

Example: *16x2 - 24x + 9*

1) Get the square root of the first and third term.

They are *4x and 3* respectively.

2) Check if the product of two and the first and second square roots is equivalent to the second term.

*(4x)(3)(2)=24x*

If this check is true, move onto the next step

3)The binomial should be written as the first square root plus the second square root if the second term was positive or the first square root minus the second square root if the second term was negative, squared.

The second term in the trinomial *16x2 - 24x + 9* is negative.  
Therefore the answer is (4x+3)2

Factoring Trinomials

In the form of x2+bx+c, a trinomial is sometimes the product of two different binomials. This means that you will have to guess and check two different numbers whose products equals c and whose sum equals b.

Example: *x2 - x - 6*

1) Think about pairs of factors of -6.

There is *1 \* -6, 1 \* -6, 2 \* -3, and -2 \* 3.*

2) Out of these pairs of factors, is any pair whose sum is the coefficient of the second term?

*The coefficient of the second term is -1.   
2 + (-3) = -1.*

3) Write out the product of binomials.  
*(x+2)(x-3)*

Sometimes there will be a coefficient attached to the first term of the trinomial, which results in *ax2+bx+c*. What you want to do first is see if there is a GCF that can be factored out. Then, if there is still a coefficient attached to the first term, the guessing and checking becomes much more complicated.

Example. *12x2+34x - 28*

1) Check for a GCF and factor it out if possible.  
  
The GCF here is 2, it should be factored out.

*2(6x2+ 17x - 14)*

2) Pick out pairs of products equivalent to c and the first term

Products of -14: *1 \* -14, -1 \*14, 7 \* -2, -7 \* 2*Products of 6 :*1x \* 6x, 6x \* 1x, 2x\*3x, 3x\*2x*

3) The rule is that one factor from a product pair of -14 multiplied by one factor from the product pair of 6 is one of the addends that makes up the sum of b. The other product of factors is the other addend.

*7 \* 3x + -2 \* 2x = 17x*

4) Write out the binomials. Each binomial must contain one variable and one of the constants and they must not have been multiplied by each other when you checked if the sum of the products was the second term of the trinomial.

*(3x-2)(2x+7)*

Factor By Grouping

Factoring *ax2+bx+c* trinomials by grouping involves a little bit of guess and check as well.

First, multiply the first and the third term, and ask yourself what factors of this product have a sum that is equal to the second term.

The equation is then rewritten as (*ax2+factor1) + (factor2 +c)*.

Then ask yourself, what common factor is shared by both groups and factor them out. The expression can then be rewritten as grouped factors.

Example: 5x2 + 18x + 9.

1) What factors of the product 5x2 and 9 have a sum of 18x.   
Product is 45x2. The sum and products of 3x and 15x are 18x and 45x2. The equation is now (5x2 + 3x)+ (15x + 9).

2) The common factor shared by both groups is 5x+3

*(x)(5x+3) + (3)(5x+3)*

3)Group the factors

*(x+3)(5x+3)*

Tip: Make sure you properly keep track of any negative signs. If you are unsure about your answer with negatives, just multiply the two binomials and see if you get the regular trinomial back.

Factoring Using Several Methods

Sometimes you will be presented with a large algebraic expression that cannot be factored in one step. You will want to follow a certain order to solve it.

1) Factor out the GCF  
2) Factor using the difference of squares  
3) Factor perfect square trinomials  
4)Factor general trinomials   
5)Factor by grouping.

Example: *2a2 - 16b2 - 4ab - 32bc + 8ac*

1)Factor out the GCF  
*2(a2 - 8b2 - 2ab - 16bc + 4ac)*

Tip: You might have to rearrange terms in order to see trinomials clearly. If you look at this expression, you might be able to see that the first three terms can be rearranged into a trinomial.  
  
This expression can be rearranged into *2(a2 - 2ab - 8b2 - 16bc + 4ac).*

*2)* There are no difference of squares or perfect square trinomials, so we move onto factoring general trinomials.

*2((a-4b)(a+2b) + (- 16bc + 4ac))*

3) Factor by grouping. In this case, a - 4b exists in both groups.

*2((a-4b)(a+2b) + (-4c)(4b-a))  
2((a-4b)(a+2b) + (4c) (a-4b))  
2((a-4b)(a+2b+4c))  
2(a-4b)(a+2b+4c)*

Solving Absolute Value Equations

The goal is to get the absolute value by itself. Once it is by itself, you can get rid of the side with the absolute value by adding a ± symbol to the other side and solving with that.

Example: *2x +* ***|****3x + 1****| =*** *12*

1) Move all terms except the absolute value to the other side of the equation.

***|****3x + 1****| =*** *12 - 2x*

2) Get rid of the absolute value by adding the ± symbol to the other side.

*3x + 1* ***= ±(****12-2x)*

Tip: You can treat the ± sign as two different equations,   
*3x + 1* ***= (****12-2x) 3x + 1* ***=*** *-****(****12-2x)*  or wait until the end to work with the plus minus symbol, *3x* ***=*** *-1*  ***± (****12-2x)*

3) No matter which path you choose, you end up having to solve for *3x* ***=*** *-1*  ***+ (****12-2x) and 3x* ***=*** *-1*  ***-(****12-2x).*

After a little bit of algebra, the result is x={-13,11/5}

Solving Quadratic Equations

Quadratic equations can be solved by factoring, solving the square, or the application of the quadratic formula.

To solve quadratic equations by factoring, all terms have to be move to such that one side of the equation must be 0. The other side of the equation should then be expressed as products of terms or groups of terms. Since the product is 0, each factor can be 0, so each group of terms has to be solved for 0.

Example: *x2+7x+11 = 1*  
  
1) To make the right side of the equation 0, 1 must be subtracted from both sides of the equation.

*x2+7x+10 = 0*

2) Turn the trinomial into a product of binomials.  
(x+5)(x+2) = 0

3) Solve each binomial for 0

*x+5 = 0 x=-5  
x+2 = 0 x= -2*

x={-2,-5}

Quadratic equations can also be solved by a method called solving the square. This method is best illustrated with an example.

Example: *2x2 + 8x + 12 = 16*

1) Solving the square only works when the coefficient of the first term is 1, so divide the entire equation by 2.

*x2 + 4x + 6 = 8*

2) Move the third term of the trinomial to the other side.  
*x2 + 4x = 2*

3) Add to both sides of the equation (b/2)^2 where b is the coefficient of the second term of the trinomial.

*x2 + 4x + 4= 6*

4) The left side of the equation will be a perfect square trinomial. Convert it into its binomial form.

Tip: The constant in binomial form is just b/2.

(x+2)2 = 6

5)Square root both sides

x+2 = ±

Tip: Don't forget the ± symbol since it technically has two roots

6) Solve the equation

x= -2±

Factoring with the quadratic formula is simple, but easy to mess up the basic arithmetic.   
Given ax2+bx+c=0, x will always equal .Do not forget about the plus or minus symbol.

The b2 - 4ac is called the discriminant. If this value is negative, x will have imaginary values.

Example: *x2 + 4x + 6 = 8*

1) This equation only works if one side has a value of 0. We must move 8 to the left side of the equation.

*x2 + 4x - 2 = 0*

2) Plug in the values for the quadratic formula.

3) Solving the equation results in

4) Expression the radical in simplest form if possible

5) Simplify more

x= -2 ±

Solving Higher Degree Polynomial Equations

You will encounter equations with polynomials whose powers go above 2. Usually, in algebra 2, these equations can be solved by making one side of the equation 0, then by factoring the other side as much as possible.

Example: *2x4=26x2 - 72*

1) Set the equation equal to 0

*2x4 - 26x2 + 72 =0*

2) Factor out the GCF and divide the equation by it

Factoring out the 2 results in *x4 - 13x2 + 36 =0*

3)The expression on the left can technically be treated and factored as a trinomial.

*(x2 - 9)( x2 -4) = 0*

4)Factor the differences of perfect squares.

*(x-3)(x+3)(x+2)(x-2) = 0*

5) Solve for x

x= {±2, ±3}

Solving Rational Equations

Rational equations in the context of algebra 2 means there will be algebraic equations with fractions that contain variables in the numerator and denominator.  
The first thing you want to do is to take a note of what the variable cannot equal. If the denominator of any fraction in the equation contains a variable, set an equation where the denominator is on one side and solve for 0. The answer to the original equation cannot be whatever you solved for.  
The second step is to factor completely every numerator and denominator.  
The third step is to compress each side of the equation into a single fraction.  
The fourth step is to solve for x as if it were a higher degree

polynomial equation.

Note that dividing equations by x is a bit dangerous and might give an entirely wrong answer. It is safer to expression the terms of the equation with x as lowest common denominator.

The fifth step is to double all your answers via substitution because you might get extraneous roots of the equation, ie. , roots that do not work with the equation

Example: = +

1) Note what x cannot equal due to it being present in denominators.

x cannot equal 4 or -4

2) Factor each numerator and denominator completely

= +

3) Compress each side of the equation into a single fraction.

= +

= +

=

4) Solve the equation using the lowest common denominator.

The LCD here is (x+4)(x-4). Only the left side of the equation needs to be converted into this form.

=

=

x2 + 4x = 2x + 8

x2 +2x - 8 = 0

(x+4)(x-2) = 0

x={-4,2}

However, x cannot be -4 since that would make 0 in the original equation.  
x cannot be 2 either because when it is plugged into the equation, the two sides of the equation are unequal.  
Thus there is no solution.

Solving Radical Equations

Radical equations are equations where the variable under a radical symbol. The first step to solving such an equation is to isolate one of the roots, then squaring, cubing, or whatever the index of the radical symbol is, both sides. If there are still radical symbols with variables under them, repeat the previous step. Then solve the equation normally. You may get multiple answers, and some of the answers will be extraneous roots, so make sure to plug the numbers back into the equation

1) Isolate one of the roots

2) Raise each side to the power of the index of the isolated radical symbol.

3) Isolate the radical symbol again

4) Raise each side to the power of the index of the isolated radical symbol.

52=k  
25 = k

Plugging 25 back into the original equation results in -1 = -1, so it is the correct answer

Solving Linear Inequalities

Solving linear inequalities is similar to solving equations, with the only rule that dividing or multiplying the inequality means that the inequality must be flipped.

Example: -2x+2 > 9

1)Subtract 2 from both sides.

-2x > 7

2) Divide both sides by -2. Since that is a negative number, the sign must be switched.

x< -3.5

If you were to plot this on a number line, you would put an open circle above - 3.5 and draw an arrow going left. If the symbol was the less than or equal to symbol, then you would put a closed circle.

If you were to write the solution in interval notation it would be (∞,-7).

Solving Compound Inequalities

Solving compound linear inequalities just involves inequalities linked by AND or OR. In inequalities joined by AND statements means that the solution set is the intersection of the two inequalities. In inequalities joined by OR, the solution set is the regular inequalities plus the overlap.

x > 7 and x < 10

Note that the and symbol is sometimes written as an arrow pointing up.  
  
The answer is 7 < x < 10, or (7,10) in interval notation.

The solution set of x > 10 or x > 12

Note that the or symbol is sometimes written as a downwards facing arrow.

The solution set is x > 10 since any number greater than 12 is already greater than 10.

Compound inequalities are sometimes written out as a < x < b, and you would have to solve for x. In any case, whatever is done to one "side" of the inequality must be done to all other "sides" of the inequality.

Solving Absolute Value Inequalities

There are rules to solving absolute value inequalities: Note that in the rules 1 to 4, wherever you see a < or >, assume that ≤ and ≥ also apply.

Rule 1) If |a| < b and b is a non-zero positive number, then

-b < a < b

Rule 2) If |a| > b and b is a non-zero positive number, then   
a < -b or a > b

Rule 3) If |a| < b and b is a negative number, then there is no solution because the absolute value of an equation must be 0 or higher.

Rule 4) If |a| > b and b is a negative number, then all solutions are true because the absolute value of any number is always at least 0.

Rule 5) If |a| < 0 then there is no solution, but if |a| ≤ 0, then a can only be 0

Rule 6) If |a| > 0, then all values of a except 0 are valid. If |a| ≥0, then all solutions are valid

Example: |x + 1| < 5  
  
1)Get rid of the inequality by using the rules.

-5 < x+1 < 5

2) Solve for x   
-6 < x < 6

Quadratic Inequalities

To solve quadratic inequalities, make one side of the inequality 0. Then factor the other sides and solve for 0. Plot the roots on a number line and test regions for where the inequalities hold true.

Example: x2 -x -5 < 1

1) Make one side 0

x2 - x - 6 < 0

2) Factor the other side

(x-3)(x+2) < 0

3) Plot the roots 3 and 2 on a number line



4) Test the locations, meaning with numbers less than -2, between -2 and 3, and greater than 3.

Testing with -3: (-3-3)(-3+2) < 0 false  
Tip: Test with 0 if possible  
Testing with 0 : (0-3)(0+ 2) < 0 true  
testing with 4 (4-3) (4 + 2) < 0 false

The only region that is true is between -2 and 3, so the solution is -2 < x < 3.

Rational Inequalities

To solve inequalities that involve rational expressions, move all the rational expressions to one side such that the other side is 0. Factor each numerator and denominator to make it easier to combine the entire expression into a single fraction. Each factor then must be solved for 0 to obtain the roots of the equation. Note that any roots of 0 obtained from the denominator of the equation cannot exist, otherwise it would make the denominator undefined. The last step is performing sign analysis to figure out what inequalities are true

Example: ≥ 0

1) Since the fraction is already on one side, the other side is already equal to 0, all we need to do for this step is to factor it.

≥ 0

2) Solve for each factor being equal to 0, results in -2,-1,-4, and 4. Since -4 and 4 were roots obtained from the denominator, the answer cannot contain -4 and 4.

3) Within each row except the last one, under a number that satisfies the inequality from the top of the column, applies to the expression in that row, would the result be positive or negative?

Then for the last row, multiply the signs of all the numbers in that column

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | < -4 | -4 to -2 | -2 to -1 | -1 to 4 | > 4 |
| X+4 | Neg (-) | + | + | + | + |
| X+2 | - | - | + | + | + |
| X+1 | - | - | - | + | + |
| x-4 | - | - | - | - | + |
| Resulting Sign | + | - | + | - | + |

The equation asks for any conditions that are greater than or equal to 0, so the solution set of x is

(-∞,-4] U [-2,-1] U [4,∞)

Recall that -4 and 4 cannot be possible answers, which means that they cannot be included in the solution set.

The end result is (-∞,-4) U [-2,-1] U (4,∞)